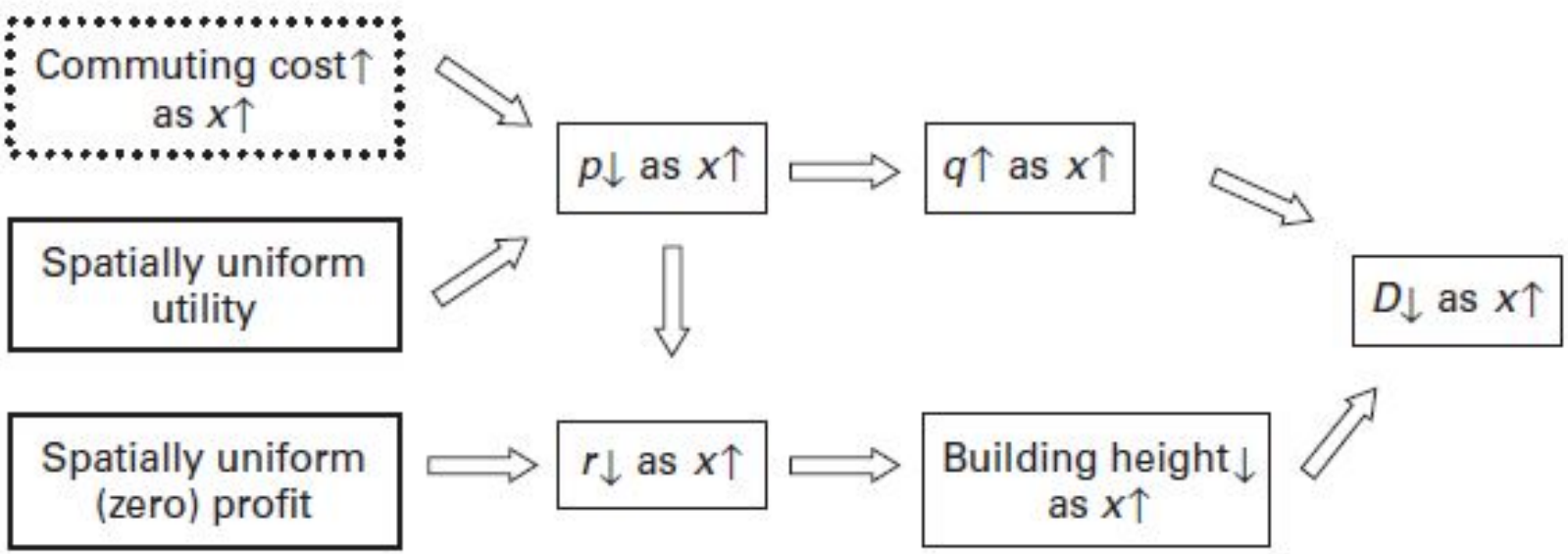
Socio-Economic Aspects of Urban Development

Unit 4: Using the Urban Economic Model for Empirical Analysis

Prof. Melanie Krause, Ph.D.

Winter Semester 26/27

Summary of the Alonso-Muth-Mills Model



Summary of the Alonso-Muth-Mills Model from Unit 3

With increasing distance x to the city center...

1. the rental price $p(x)$ decreases
2. floor space per person $q(x)$ increases
3. the leasing rate for land $r(x)$ decreases
4. building height $S(x)$ decreases
5. population density $D(x)$ decreases

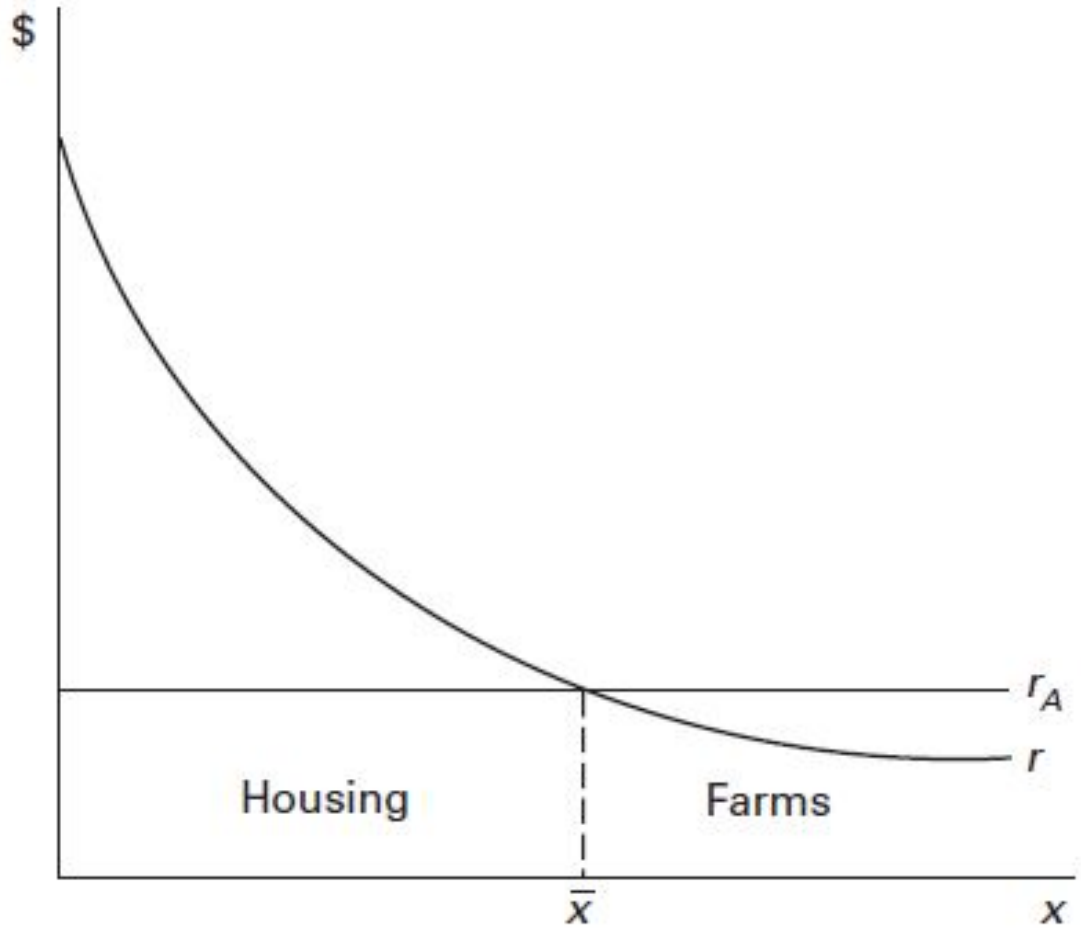
- In this unit, we are going to use the model to
  - analyze the adjustments that happen when a shock occurs (as in the textbooks by Brueckner (2011) and O'Sullivan (2011))
  - compare model predictions with reality (as in various research papers)

Where are the City Boundaries?

- To close the model, we can calculate where the city boundaries are located.
- We have a supply-demand equilibrium of the city: As much housing is supplied as demanded.
- Construction firms and farmers compete for the land:
  - Construction firms pay the leasing rate $r(x)$
  - Farmers pay an agricultural rate r_A (reflecting agricultural productivity, independent of x)
- $\Longrightarrow$ Locations with $r_A < r(x)$ will be construction land, locations with $r_A > r(x)$ will be farmland.

Where are the City Boundaries?

The city boundaries are located at $\bar{x}$, where the leasing rate that construction firms are willing to pay equals exactly the agricultural rate: $r_A = r(x)$.



Comparative Statics

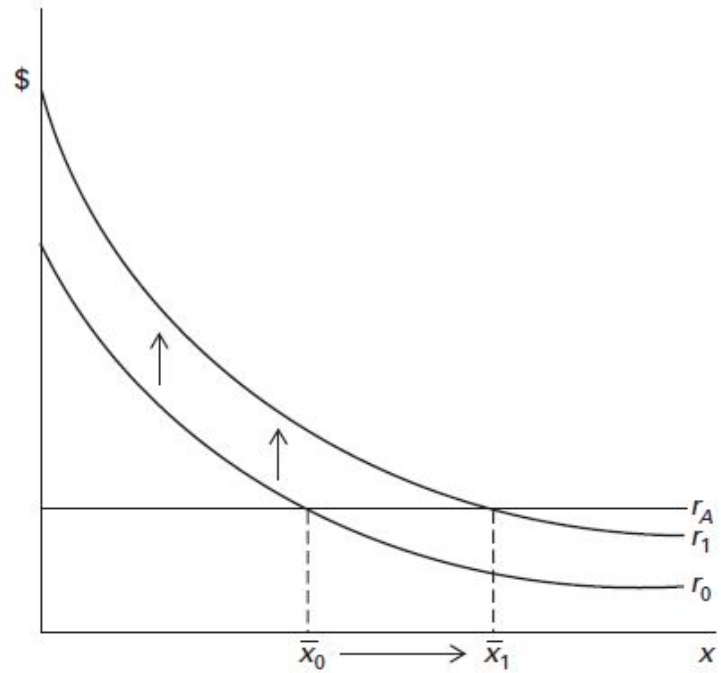
- Supply-demand equilibrium: The population N must fit exactly into the housing available inside the city boundaries at $\bar{x}$.
- The urban equilibrium depends on the four key parameters of the model:
  - population N
  - agricultural rent r_A
  - commuting cost τ
  - income y
- Let us change one parameter and think about the adjustment process!

Increasing the Population N

Start out in equilibrium. The population N increases (e.g. due to migration):

1. Excess demand for housing
2. Rental price p per square meter increases everywhere in the city
3. With income kept fixed, residents choose smaller apartments, q decreases
4. Thanks to higher p, construction firms compete more vigorously for land, driving up the leasing rate r at all locations
5. With higher r, construction firms economize on land in the building process and build higher buildings everywhere
6. Higher buildings and smaller apartments lead to higher population density D everywhere
7. Because of higher r, the urban land-lease curve shifts up: $r(x) > r_A$ at more locations. The city boundaries shift outwards
8. With higher density and more area, the city grows until fitting the higher population N

Increasing the Population *N*



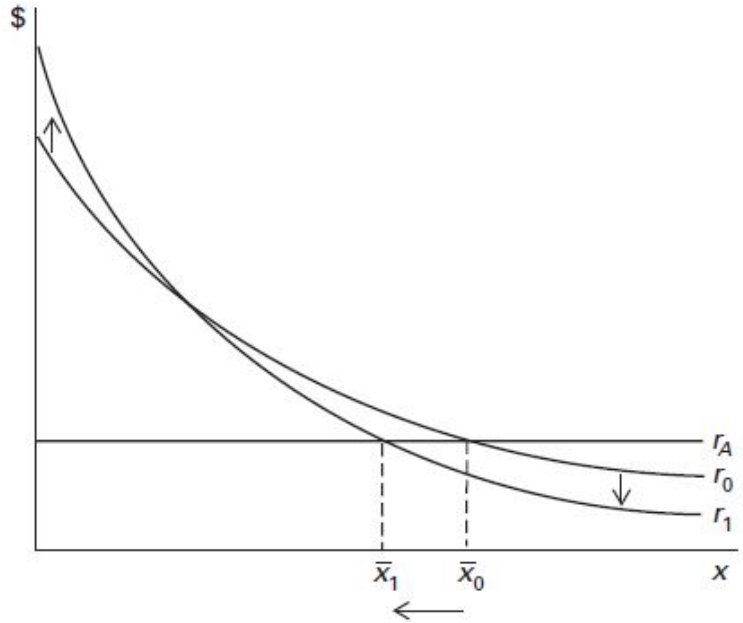
Note when interpreting: Some features of the adjustment process - decreasing the apartment size, building taller buildings - refer to the long run. Instead of seeing them as changes over time in one city, it makes more sense to consider them as a **comparison between two cities**: Empirically, more populous cities tend to have smaller apartments, more high-rise buildings and higher population densities everywhere compared to less populous cities.

Increasing the Commuting Cost τ

Start out in equilibrium. The commuting cost τ increases (carbon tax?):

1. For commuters on the outskirts, the lower rental prices do not provide an adequate compensation for long suburban commutes any more.
2. Demand for housing increases in the inner city, so that $p \uparrow$, and decreases on the outskirts, so that $p \downarrow$. The rental price curve rotates clockwise.
3. Construction firms follow the development of rental prices p, competing more strongly for plots in the inner city ($r \uparrow$) and less strongly on the outskirts ($r \downarrow$). The land-lease curve rotates clockwise.
4. $r(x) < r_A$ at some locations, so that the city boundary decreases from $\bar{x}_0$ to $\bar{x}_1$. The city area shrinks.
5. Demanded floor space q decreases in the inner city as $p \uparrow$. For q on the outskirts the effect is ambiguous: Budgets are smaller with higher τ, but the rental price p has decreased.
6. Density increases in the inner city with smaller q and taller buildings. On the outskirts, it is ambiguous as the change in q is not clear.

Increasing the Commuting Cost τ



Brueckner (2011) compares average European cities (high gasoline prices) to average American cities (low gasoline prices):
More compact cities, smaller land area, taller buildings and smaller apartments in the center, higher rental prices, higher land lease rates, higher population density

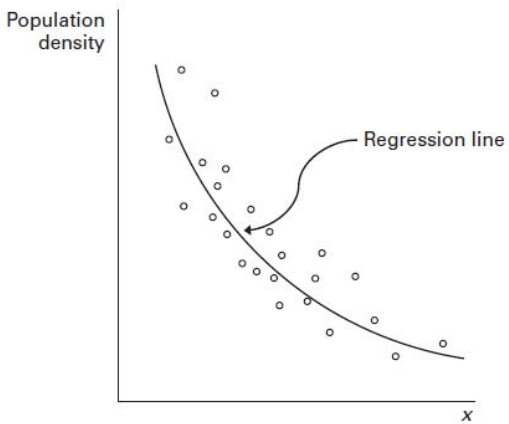
What is Missing in the Urban Economics Model?

- A model is always a simplification of reality and it is obvious that some (important?) characteristics of cities do not feature in the simple Alsonso-Muth-Mills model.
- Class discussion: Take a moment to reflect:
  - Name two or three things that you think are important aspects/issues of urban life and which you cannot find in the simple Alsonso-Muth-Mills model.
  - Do you think that including these aspects into the model would alter the model predictions of, e.g.
    - ★ population density as a decreasing function of x?
    - ★ building height as a decreasing function of x
    - ★ rental prices as a decreasing function of x?
  - Or will these predictions hold nevertheless?

Population Density Gradients in 57 World Cities

- Bertaud and Malpezzi (2014) were the first to conduct an empirical study: *"The Spatial Distribution of Population in 57 World Cities: The Role of Markets, Planning, and Topography"*
  - They examine how well the model prediction of a decreasing population density works in reality. Is population density is an exponential function depending on x?

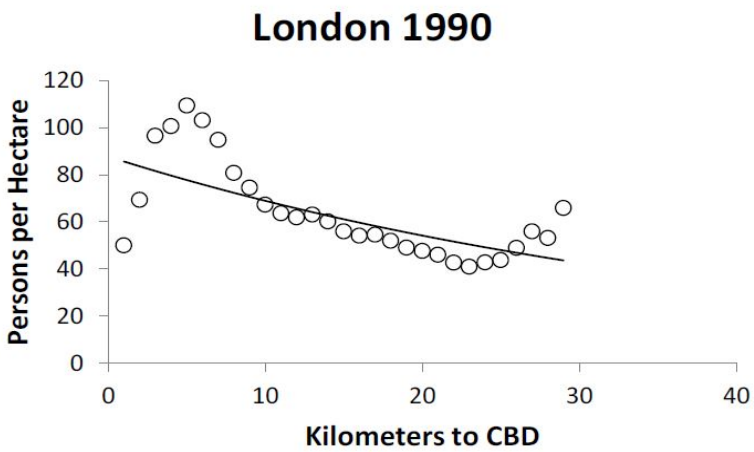
$$D = \alpha \cdot exp(-\beta \cdot x) \tag{1}$$

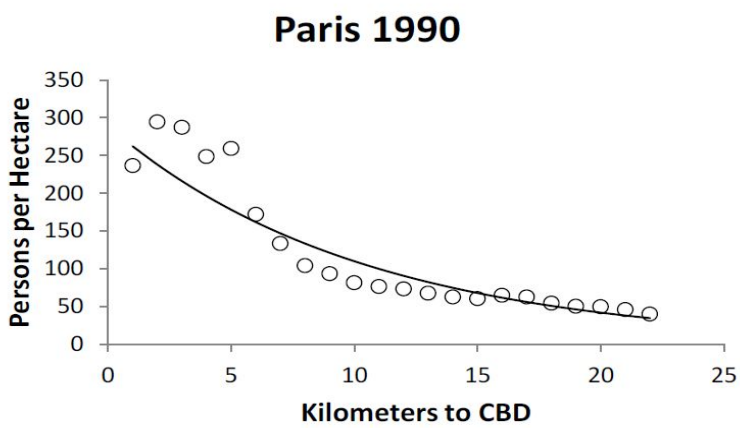


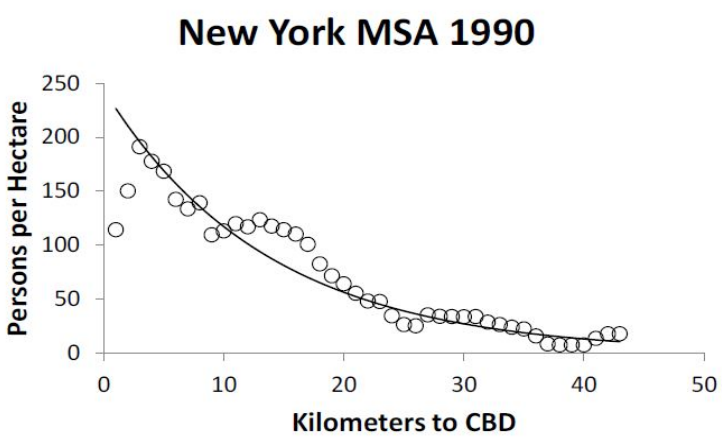
Population Density Gradients in 57 World Cities

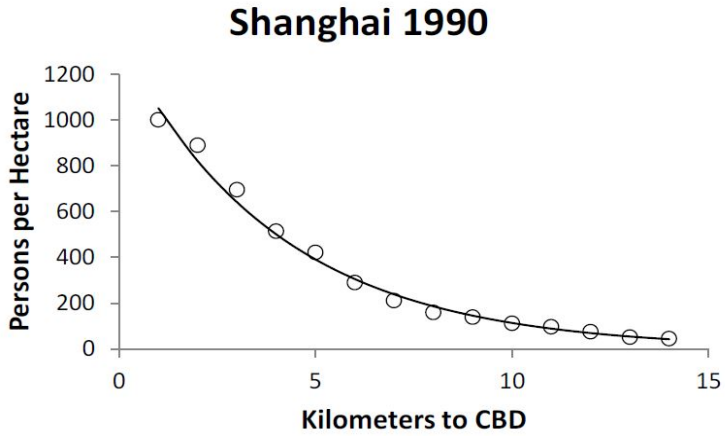
- Bertaud and Malpezzi (2014) obtain census tract level population data from 57 cities around the world.
- They put rings (annuli) of 1 km width around the city center of each city.
- They map the population from the census tracts to the rings. With population density within each ring, they can analyze how quickly it decreases with distance from the city center.
  - Where is the city center? Employment center plus traditional local opinions of the city center (such as city hall)
- β, the **population density gradient**, captures the rate at which population density falls from the city center. Population density is measured as people per hectare of built-up area.
- Let us look at the gradient plots of some selected cities. Do we observe an exponential decay - or do some cities deviate from the pattern of decreasing population density predicted by the model?

Population Density Gradients in 57 World Cities

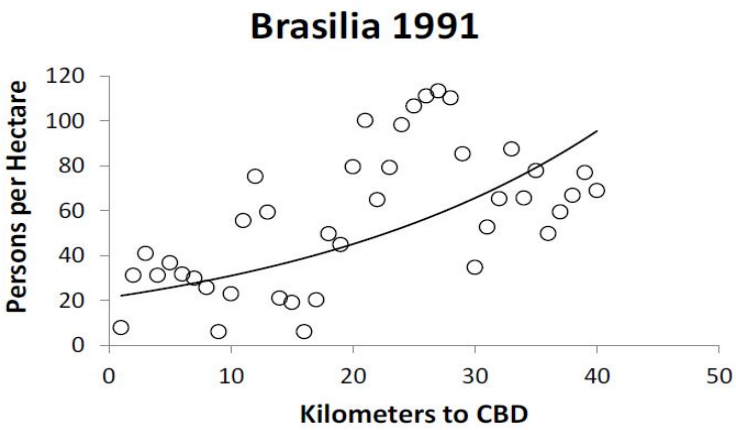


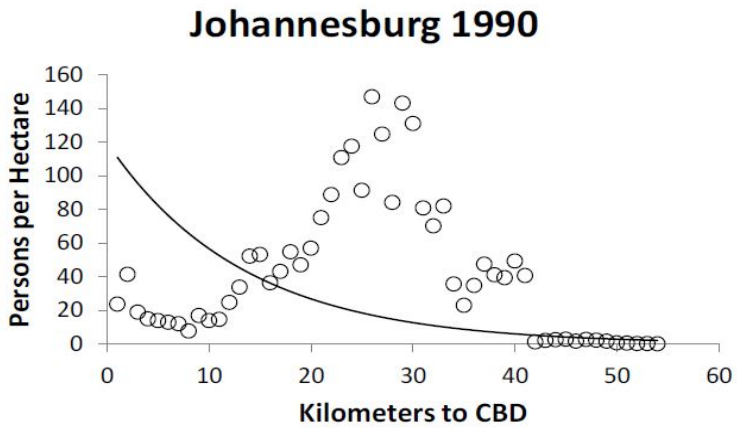


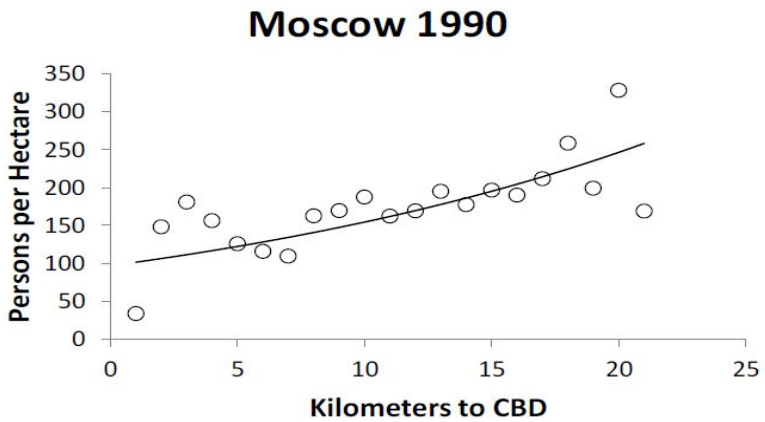




Population Density Gradients in 57 World Cities







Population Density Gradients in 57 World Cities

- The gradient plots show that some population gradients are steeper than others - and in particular cases, such as Moscow, even positive
- Taking logs allows us to estimate a linear regression of log population density on distance x:

$$D = \alpha \cdot exp(-\beta \cdot x) \quad (2)$$

$$log(D) = log(\alpha) - \beta \cdot x \quad (3)$$

- The main focus is β, the **population density gradient**:
  - Population density gradients range from -0.489 (Guangzhou) to 0.047 (Moscow). In line with the model, most cities have negative gradients so that population density decreases with distance to the center.
  - Cities with steep, clearly negative gradients have the highest $R^2 \Longrightarrow$ Exponential function fits very well.
  - Average population gradient of -0.1563 (in cities with R^2 above 0.9).

Population Density Gradients in 57 World Cities

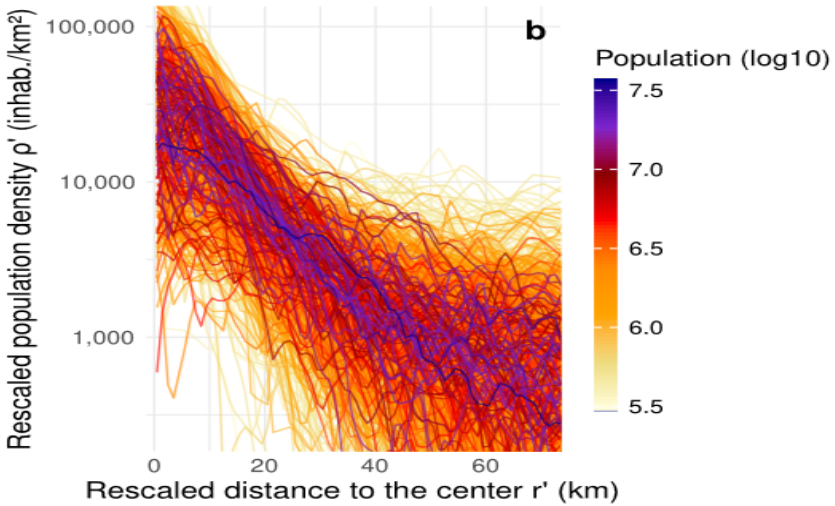
- More populous cities tend to be more decentralised, with flatter gradients
- Population density gradients are steeper in European than in American cities, as they are less extended and less car-dependent
- The maximum densities also vary by city: In its densest location, London has only one half the density of New York and one third of that of Paris.
- Why is the maximum population density often not directly in the city center but close by (2nd or 3rd ring)?
  - Offices and shops rather than residential buildings tend to dominate directly in the center.

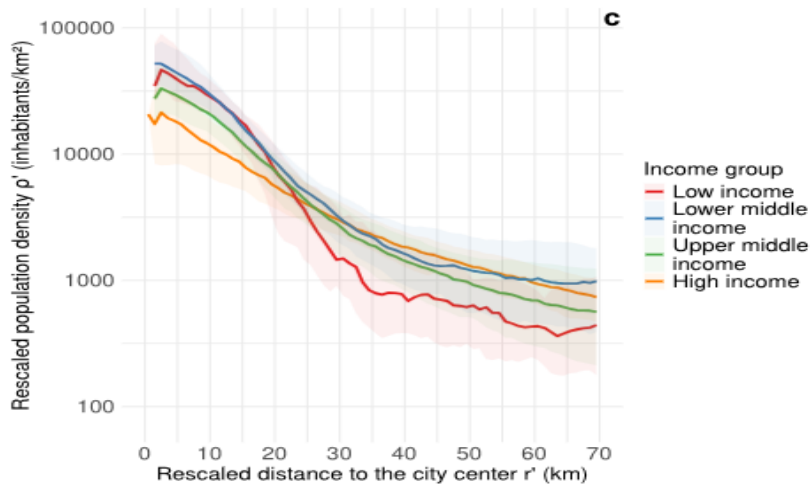
Population Density Gradients in 57 World Cities

- What is (or *was*) going on in ...
  - Brasilia?
    - ★ A planned city created from scratch. Planned for a population of 400,000, it has attracted many more residents that live in satellite towns on the outskirts.
  - Johannesburg?
    - ★ Still under apartheid in the early 1990s with the corresponding land use regulations. The traces remain visible in the city structure afterwards as well $\Longrightarrow$ Very irregular spatial structure
  - Moscow?
    - ★ Housing development by Soviet kombinats without market information.
    - ★ Putting up new housing on the edge of the city, instead of tearing down obsolete factories and recycling land $\Longrightarrow$ Creating satellite towns with extensive commuting times on public transport, leading to positive population density gradients

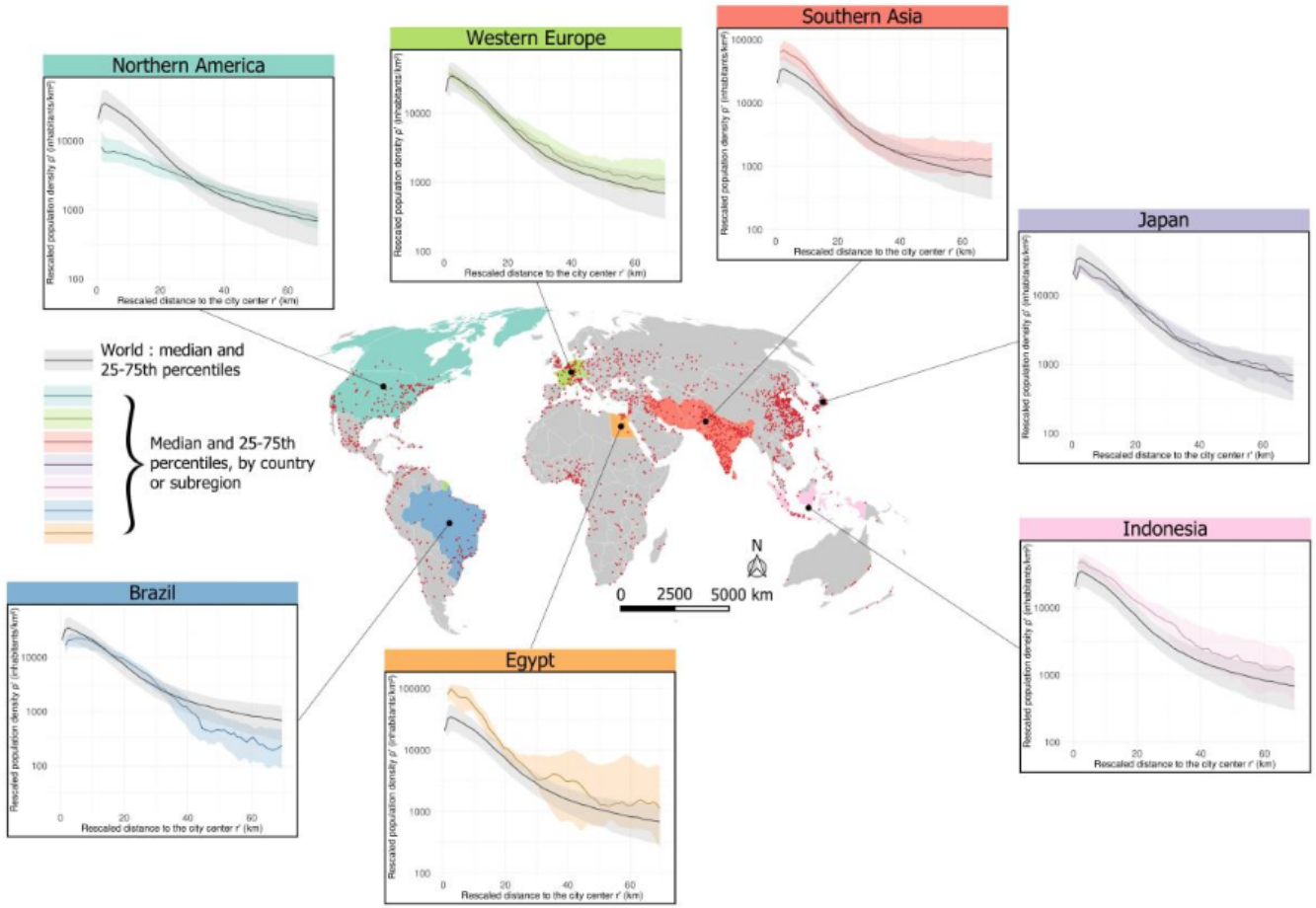
Laziou and Lemoy (2025): Population Density Gradients

- Building on Bertaud and Malpezzi (2014), Laziou and Lemoy (2025) use 2020 data on population and built-up area from the Global Human Satellite Layers ($\rightarrow$ Unit 5) for a newer and more extended analysis
- **Population density gradients for 1,593 cities from around the world!**
  - Obviously heterogeneity in patterns, but note the clearly predominant downward slope. And flatter for cities in high-income than low-income countries.





Laziou and Lemoy (2025): Population Density Gradients



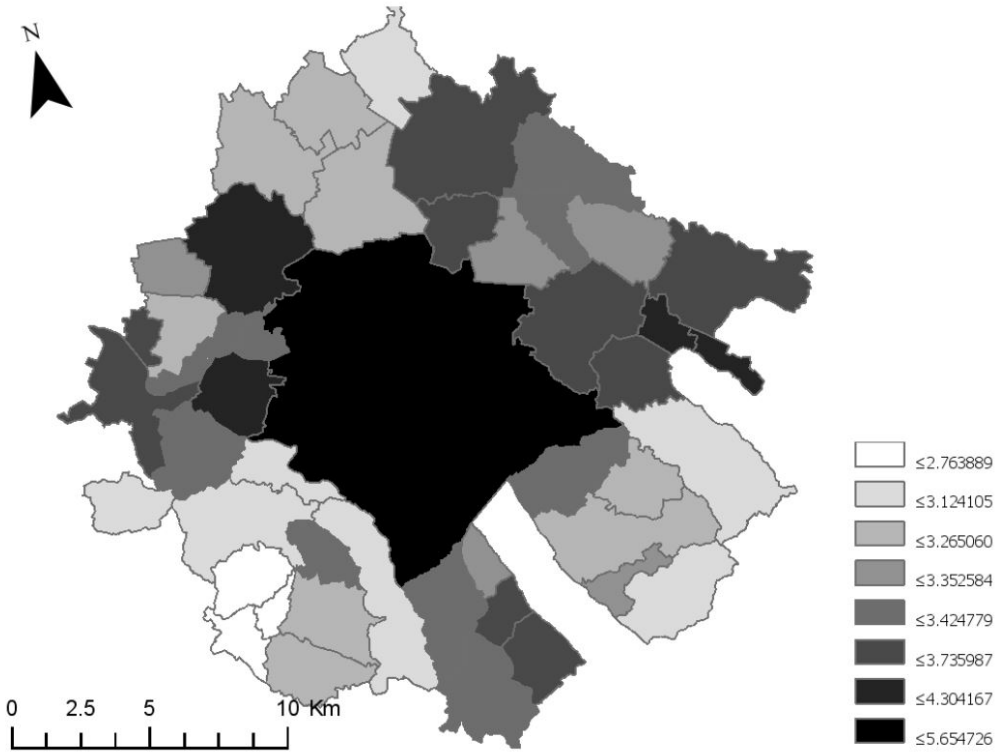
- Note the less regular patterns and larger interquartile ranges (more heterogeneity) in Egypt and Brasilia than in the US and Japan

Building Height and Rental Prices

- What about the other predictions of the Alonso-Muth-Mills model in reality?
- **Liotta et al (2022)** compile fine-grained data on **population, rents, housing sizes, and transportation costs in 192 cities** worldwide (using data from Google Maps and real estate websites):
  - In 167 of the 192 cities, rental rates per square meter decrease with distance to the city center.
  - The geographic extent of cities tends to increase with population and income and decrease with agricultural rent and transportation costs.
  - Coastal location, polycentricity, and informal housing/'slums' provide deviation from model predictions. Nevertheless, the model fits surprisingly well for most cities worldwide, the authors conclude.
- A number of other papers look at individual cities - such as **Danton and Himbert (2018)** on **Zurich**.
  - Even without computing gradients, one can see the spatial pattern of building height and rental prices (years 2002-2014) in locations closer to and further away from the city center

Building Height and Rental Prices: Zurich

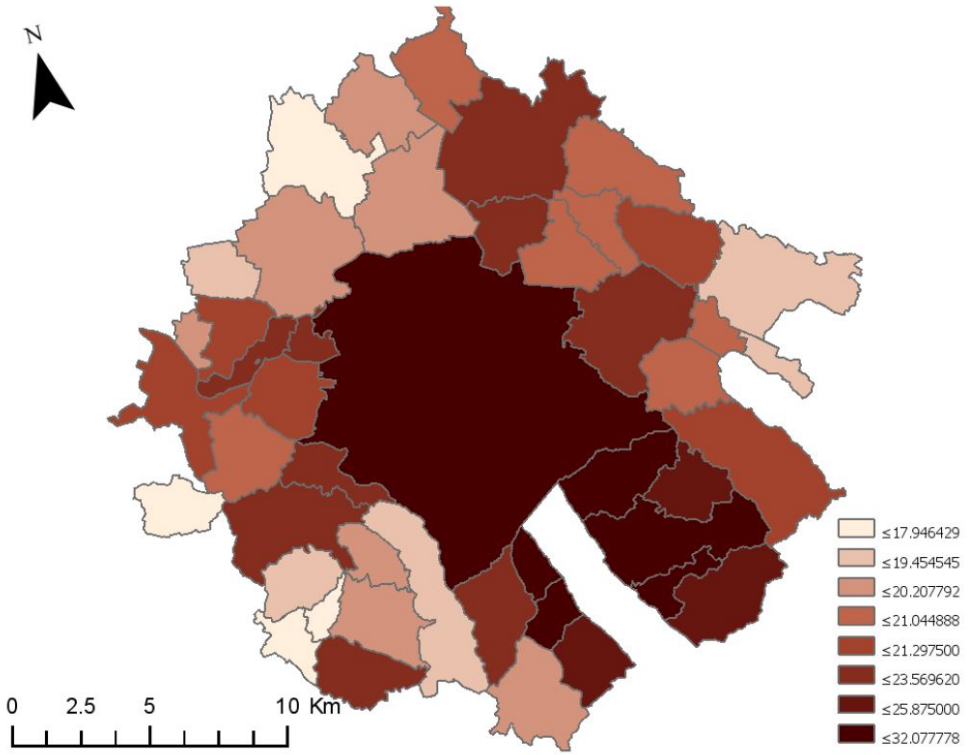
Figure 1: Average Building Height (Floors) Variation across Locations in Zurich*



As one moves away from the city center (around the central station and business district), building height decreases.
Note that it decreases less steeply (darker rather than lighter shades of gray) on the North-Eastern part of the city - maybe because there is the city of Winterthur and thus a second city center nearby?

Building Height and Rental Prices: Zurich

Figure 2: Average Rental Price (CHF/m^2) Variation across Locations in Zurich*



Rental prices per square meter clearly decrease with distance to the city center.
Exception: Rental prices stay high in the South East of the city - at the shores of the lake!

Limitations of the Model / Factors Influencing the Gradients

- We have seen that the Alonso-Muth-Mills model tends to capture important stylized facts about rental prices, apartment space and population density which are in line with the structure of many cities.
- What can the model not capture? Where does it fall short?
- Think about the class discussion, as well as
  - Geographical features
    - ★ Seaside, rivers, mountains but also parks and green spaces.
    - ★ Geography decreases available building land but also provides amenities (ocean view!) $\Longrightarrow$ Potentially less regular patterns
  - Cultural and historical amenities
    - ★ Are all locations equally desirable living areas? Or do some offer better schools, restaurants, theaters, historic buildings than others? This can enter the tradeoff between distance, rental price and commuting costs. $\Longrightarrow$ If city center locations have desirable amenities, the gradients might be steeper.

Limitations of the Model / Factors Influencing the Gradients

- Further Factors:
  - Income heterogeneity
    - ★ The simple model cannot tell us where in the city rich and poor people will live. We will consider model extensions that allow for income heterogeneity in the context of income inequality and segregation.
    - ★ How will income heterogeneity affect the gradients? If rich people live on the outskirts, rental price gradients will be less steep (maybe even positive?).
  - No difference between renting or buying an apartment in the model
    - ★ One can compute price gradients for both with empirical data, one might be steeper than the other.
  - What about a polycentric city structure? Secondary business district on the outskirts?
    - ★ Potentially less regular gradients, increasing again when approaching the second center?
  - Urban planning, political/institutional aspects
    - ★ Moscow's satellite towns are an extreme example of upending the model predictions and obtaining a positive population gradient

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